# Trends in Research in Problem Solving in Mathematics

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**Abstract: This study explored trends in problem solving in mathematics research from 2000 to 2020. A total of hundred (100) articles related to mathematical problem solving were examined from the selected journal articles available online. Topical issues as well as methodological trends (research methods, type and size of sample, tools for data collection and techniques for data analysis) were investigated. Content analysis was employed to analyse the sampled data using open codes according to the categories, and themes were generated for some research questions. The results showed that topical issues mainly investigated in mathematical problem solving within the trend period were problem solving ability, teaching, meta-cognition, thinking through, views, process, strategies, creativity, difficulties, efficacy, assessment, performance, representation, and problem-solving curriculum respectively. Methodological trends showed a preference for the qualitative research method. Tests and interviews were the dominant data collection tools utilized while qualitative descriptive analysis was the most preferred data analysis technique. It is therefore concluded that this study can serve as a foundation for further studies and an enhanced study on mathematical problem solving.**

**Keywords: Trends, Research, Problem Solving, Mathematics**

## 1. Introduction

Man encounters many problems in life which require solutions. A central objective of education is the training of an individual on how to solve the numerous problems faced in life and to become a problem solver (Yavuz, Deringol-Karatas, Arslan & Erbay, 2015). Thus, problem solving skill is an essential objective in the education of today (Inter Ponce, 2020).

In the past, any mathematics activity has been described as problem solving. Many erroneously believed that “mathematics is synonymous with solving problems – doing word problems, creating patterns, interpreting figures, developing geometric constructions, proving theorems, etc.” (Wilson, Fernandez & Hadaway, 1993). This portrays mathematics as a subject of repetition which elevates memorization of facts as being essential to its learning. As reported by Kilpatrick (1969) as cited in Inter-Ponce (n.d), problem solving research around the 70s focused on actual solution to the problems or answers to the exercises.

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In later years, a shift in orientation was observed in the main aim of mathematics teaching and learning to the development of ability to tackle various problems that are complex in mathematics (Wilson, Fernandez & Hadaway, 1993). Emphasis was placed on students' learning with comprehension, understanding of what they were learning, and ability for critical and creative thinking in solving problems (Allevato & Onuchic, 2008).

According to English, Lesh and Fennewald (2008):

> "Existing, long-standing perspectives on problem solving have treated it as an isolated topic where problem solving abilities are assumed to develop through initial learning of concepts and procedures followed by practice on "story problems", then through exposure to a range of strategies (e.g., "draw a diagram", "guess and check"), and finally, through experiences in applying these competencies to solving "novel" or "non-routine problems" (p. 46).

Currently in mathematics, most educators have placed a great focus on problem solving research based on their beliefs that it is a meaningful and important kind of learning and thinking (Jonassen, 1997). It is recognized as an advance thinking ability with various thinking processes useful in solving problems in any field and day to day life. It is also viewed as both a goal of learning mathematics and an essential tool of teaching mathematics (Akyuz, 2020). Unfortunately, this research is yet to correspond with the rapid changes in mathematical problem solving required after school (English, Lesh & Fennewald, 2008). A decline in problem solving researches conducted in the past decade was reported to be caused by the following factors including: cyclic trends in educational policy and practices that are discouraging, inadequate concept development and problem solving researches, inadequate students' problem solving knowledge beyond the classroom, problem solving types and mathematical thinking changing nature required beyond school and deficient problem solving accumulation research (Lesh & Zawojewski, 2007 as cited in English, Lesh & Fennewald, 2008). There is therefore the need to study the trend of research in problem solving to view what research in problem solving was in the past twenty years and offer some directions on how the field may be advanced.

In this paper, problem solving in mathematics is documented which includes: what problem is, kinds of problems, what problem solving is, problem solving instruction approaches, processes of problem solving and why problem solving? Thereafter, the trend analysis of journal articles published on problem solving in mathematics between the periods of 2000 to 2020 was examined.

### 1.1 What is a Problem?

Literarily, a problem implies a difficulty requiring a resolution or tackling. It could also be described as a puzzling circumstance (Wikitionary, 2020). In other words, problem is simply everything that we do not know how to do but we are interested in doing (Allevato & Onuchic, 2008). For Jonassen (1997), a "Problem is an unknown that results from situation in which a person seeks to fulfill a need or accomplish a goal" (p. 66). Van de Walle (2001) as cited in Allevato and Onuchic (2008) regarded a problem as any task or activity with no prescribed or memorized method for arriving at the correct solution.

Thus, Schoen and Oehmke (1980) as cited in Inter Ponce (2020) maintained that a task becomes a problem to an individual if:

1. the task calls for a solution under certain specified conditions.
2. the individual understands the task but does not see an immediate strategy for the solution; and
3. the individual is motivated to search for the solution.

According to Kantowski (1977) as cited in Hoosain (2004), an individual encounters a problem when confronted with an unanswerable question or unresolvable situation with the immediate knowledge except by creatively utilizing available previous information at arriving at the solution.

In the context of mathematics, a problem is a mathematical task that is problematic. That is to say, a problem is a task or a question that is complex or difficult to resolve with the immediate knowledge available, having no readily accessible algorithm that will result in the solution but requires a critical thinking in utilizing the diverse information available at arriving at the solution. A mathematical problem differs from an exercise in that the algorithm that will result in the solution in unavailable but for exercise the solver determines and manipulates the algorithm previously learnt or known to arrive at the answer of an exercise (Hoosain, 2004). Hoosain (2004) maintained further that problem is relative in that what is a problem to one might be an exercise to another. For instance, $2x+3=9$ may be a problem to a Basic one pupil but not for a Basic 7 pupil. Hence, a problem's difficulty or complexity is dependent on the solver's knowledge, experience, and disposition (Xenofontos & Andrews, 2014). Students' problem-solving skills are to be developed and acquired in schools. Thus, the problem content and complexity is a function of students' grade level and age increase (Akyuz, 2020).

### 1.2 Kinds of Problems

Diverse problems encountered by humans could be presented or discovered, well-defined or ill-defined, simple, or complex, long-term or short-term, and familiar or unfamiliar (Arlin, 1989 as cited in Jonassen, 1997).

These problems varieties are classified into three kinds namely:

1. Puzzle problems
2. Well-structured problems
3. Ill-structured problems

- Puzzle Problems

  A puzzle is a game or a problem that tests the ingenuity or knowledge of a person requiring the logical arrangement of pieces of information to solve (Wikipedia, 2020). In other words, it is a mathematical or logical problem that tests imagination, cleverness and skill of the solver (Puzzles9, 2020). Clontz (2020) viewed a puzzle as any task designed to entertain the solver which has a well-defined solution. According to Jonassen (1997), puzzles are well structured problems with one correct answer and known solution elements requiring the utilization of logical algorithmic processes. He maintained further that puzzle problems are content or domain independent requiring no background mathematical knowledge, irrelevant and not linked to school learning or daily real-world practice.

- Well-structured Problems

As described by Jonassen (1997), well-structured or well-defined problems are domain or content-dependent problems requiring the application of concepts, rules, principles, and skills previously learnt to a constrained problem situation. That is to say, they have a well-defined initial state, a known goal state and constrained set of logical operators and a set number of possible solutions which are either 100% right or 100% wrong (Jonassen, 1997; Study for success, 2017). This kind of problem is usually found in textbook chapters dependent upon constrained knowledge preceding the problem (Jonassen, 1997).

- Ill-structured Problems

These are complex, ill, and poorly defined problems having many possible alternative answers and not one best answer. Also, they require the integration of several content domains (Jonassen, 1997; Study for success, 2017). This kind of problems reflect real world problems encountered in everyday practice (Serc, 2018).

Other types of problems include:

1. Routine mathematics problems
2. Non-routine mathematics problems
3. Real problems (Akyuz, 2020; Marchis, 2012).

- Routine Mathematical Problems

They are tasking whose method, algorithm, technique, or formula needed to solve them is known as they have been previously taught or learnt beforehand. They are otherwise regarded as routine exercise by Schoenfeld (1985) as cited in Marchis (2012). They are usually found in textbooks and can be resolved via mathematical basic operations (Akyuz, 2020). In other words, they can be straightly solved by the application of available standard algorithm, formula, or procedure (Marchis, 2012 as cited in Berisha, Thaqi & Jashari, 2014).

- Non-Routine Mathematics Problem

According to Marchis (2012), these are problems that are unfamiliar to students with no available immediate algorithm to apply to resolve them. That is to say, they are tasks that do not have an immediate apparent strategy or algorithm to solve them but whose solution requires some degree of creativity or originality. They are usually solved in multiple ways. They challenge thinking skills and require various skills in resolving them including the organization and classification of data, discovering of the relations and determination of the rules and generalities (Akyuz, 2020). In other words, they are problems with no clear path to the solution and with no algorithm that can be directly applied to guarantee a solution (Inter Ponce, 2020). That is, they require creativity and cannot be solved with just the application of a known algorithm (Marchis, 2012 as cited in Berisha, Thaqi & Jashari, 2014).

- Real Problems

  These are complex real-life situation that must be resolved, oftentimes with no exact solution but optimal solution to fit the condition (Kantowsky, 1981 as cited in Inter Ponce, 2020).

**1.3 What is Problem Solving?**

Explicitly, problem solving entails the definition of a problem; determination of the cause of the problem; identification, prioritizing, and selection of solution alternatives; and implementation of a solution (The Executive Guideline, 2020). In the past, verbal or word problems were regarded as problem solving but according to Wiebe (1993, p.13) as cited in Inter Ponce (2020) "problem solving is the process of searching for and finding solutions to problems".

In mathematics, National Council of Teachers of Mathematics (NCTM) (2020) described problem solving as mathematical tasks having the potential of providing intellectual challenges that will enhance students' mathematical understanding and development.

From literature, problem solving has different conceptions. Majorly, it has been defined in two ways:

1. as a process of finding solutions to problems.
2. tasks that arouse students' thinking faculty (Allevato & Onuchic, 2008; NCTM, 2020; Wiebe 1993 as cited in Inter Ponce, 2020).

George Polya (1945) as cited in Allevato and Onuchic (2008) tagged the 'father of problem solving' emphasized the essence of discovery and the need to encourage students to think via problem solving in the book titled 'How to solve it'. Polya claimed that "A great discovery solves a great problem" and that discovery exists in the solution of any problem.

Learning with comprehension started gaining attention in mathematics instruction around the world from the end of 1970. Thus, problem solving was one of the accepted themes that enhances the teaching and learning of mathematics at the 3rd International Congress on Mathematical Education, in Karlsruhe, Germany. This resulted in the publication of a document titled "An Agenda for Action" in 1980 where it was recommended by the NCTM in U.S.A. that problem solving should be a focus to enhance school mathematics progress in the 1980s. Consequently, there was development of resources in facilitating problem solving in the classroom including problems collections, strategies listing, suggestions for activities and evaluation guidelines of students' problem-solving work which assist teachers in making problem solving as the basis of their work (Allevato & Onuchic, 2008). Currently, problem solving plays an important role in mathematics education, and becoming a good problem solver requires a knowledge base development in mathematics (Wilson, Fernandez & Hadaway, 1993).

Approaches to Problem Solving Instruction

There are basically three approaches to problem solving instruction namely:

1. Teaching about problem solving.
2. Teaching for problem solving.
3. Teaching through problem solving.

- Teaching about problem solving

  This is also referred to as theorising about problem solving. It implies highlighting or emphasizing Polya's model of problem solving. This entails the description of the 4-stage mathematical problem-solving process namely: understanding the problem, devising a plan, carrying out the plan and looking back. It also involves teaching of problem solving heuristics or strategies that can be utilized in executing solution plans such as: looking for patterns, solving a simpler problem, working backwards, making a table or chart, and so on ( Allevato & Onuchic, 2008; Schroeder & Lester, 1989).

- Teaching for problem solving

  This simply implies teaching problem solving as an instructional goal which is for usability and applicability in the resolution of routine and non-routine problems. That is to say, it entails teaching mathematical knowledge for the basic purpose of developing the ability to use it in solving problems. Students are exposed to many mathematical concepts, skills, principles and structures which they can transfer and apply in solving problems both within and outside mathematics i.e. the real world (Allevato & Onuchic, 2008; Schroeder & Lester, 1989).

- Teaching through problem solving

  Teaching through problem solving entails utilizing it as an instructional method. According to Allevato and Onuchic (2008), it was emphasized and recommended as a standard process for mathematics teaching as developed by the NCTM in the publication of Principles and Standards for school mathematics in the year 2000 because of the focus gained in research and studies in the 1990s. It is a means of learning mathematics (Schroeder & Lester, 1989). In other words, problems serve as the basis for exploring mathematics curriculum ideas, building skills and understanding, and developing flexible and capable mathematical thinkers (Oldridge, 2017). For Oldridge (2017), this implies "using problems, questions, or tasks that are intellectually challenging to arouse mathematical thinking through both mathematical content and mathematical processes in our schools." That is to say, it is an instructional approach that starts with a problem to be solved.

For a teacher to effectively teach mathematics through problem solving, the following basic components must be considered according to Van de Walle (2001) as cited in Allevato and Onichic (2008):

1. An effective mathematics teacher should develop an appreciation of the subject and be interested in "doing mathematics".
2. An effective mathematics teacher should comprehend students' learning and how they construct ideas.
3. An effective mathematics teacher should be knowledgeable in planning and selecting the appropriate tasks that should enable and enhance students' problem-solving learning.

4. An effective mathematics teacher should be vast in blending evaluation with the problem-solving process to improve learning and enhance their daily instruction.

### 1.4 Why Problem Solving?

Mathematical problem solving is important and of great value to students for many reasons which include:

1. Problem solving is the heart of mathematics.

   The art of problem solving is the heart of mathematics. Problem solving is the totality or essence of mathematics discipline. Hence, the instruction should be designed for students to experience mathematics as problem solving (Wilson, Fernandez & Hadaway, 1993).

2. Problem solving builds mathematical power.

   It equips students with the ability to think mathematically and apply their previously constructed knowledge in mathematics in other disciplines to solve problems that are hypothetical and real (Nzmaths, 2020).

3. Problem solving emphasizes understanding.

   Problem solving focuses on students' sense making of mathematical ideas within a problem context (Allevato & Onuchic, 2008; Nzmaths, 2020).

   Problem solving arouses students' interest and enthusiasm in solving mathematical problems (Wilson, Fernandez & Hadaway, 1993).

4. Problem solving is fun and enjoyable.

   It creates opportunity for students' personal work at their own pace and decision making about the way they explore problem (Nzmaths, 2020; Wilson, Fernandez & Hadaway, 1993).

Students benefit a lot from incorporating problem solving into the mathematics curriculum hence, it should be emphasized in mathematics instructions. Unfortunately, many teachers don't include problem solving activities in school mathematics instruction as claimed by Wilson, Fernandez and Hadaway (1993). Their reasons include it is too difficult; consumes times; inadequate space to accommodate it in the already full school curriculum; lack of evaluation process to measure and test its skills; lack of appropriate mathematics problems in textbooks; and the belief of mastering basic facts via drill and practice before utilizing problem solving.

### 1.5 What is Problem solving Process?

#### 1.5.1 George Polya's 4-Step Problem Solving Process

George Polya, regarded as the father of modern problem solving gave a four-step process for tackling mathematical problem of any kind (Awofala, 2002; Seward, 2011). He published the book "How to solve it" in 1945 where he identified four basic steps to solving problems (Berkeley Math, 2020; Jacobs, Martin, Ambrose & Philipp, 2014; Motter, 2012; Awofala, 2002).

The four-step problem solving process includes the following:

1. Understand the problem
2. Devise a plan
3. Carry out the plan
4. Look back

- Understand the problem (Preparation)

  This is the preparation stage where the solver reads the problem carefully to comprehend all the statement of the problem; to identify the unknown, data and condition; to restate the problem in own words and introduce suitable notation, to think of a picture or diagram that might help in understanding the problem and to gather enough relevant information that will enable the resolution of the problem (Liljedahl, Santos-Trigo, Malaspina & Bruder, 2016; Awofala, 2002).

- Devise a plan (Translate)

  This is the stage where the solver translates the problem and come up with a way or appropriate strategy of solving the problem. These heuristics or strategies include guess and check, make an orderly list, solve an equation, look for a pattern, draw a picture, solve a simpler problem, use a model, work backward, use a formula, etc. (Seward, 2011; Awofala, 2002).

- Carry out the plan (Solve)

  This is the stage of execution of the strategy or plan devised in step two in resolving the problem (Awofala, 2002).

- Look back (Verification)

  This is the verification stage where the solver checks to determine if all the information given in the problem was utilized and discover if the answer arrived at is reasonable (i.e. makes sense) (Seward, 2011). This is the stage of reflection on what have been done to determine what worked and what didn't work which will enable a prediction of strategy to utilize in solving future problems (Motter, 2012; Awofala, 2002).

**2. Research Questions**

The main purpose of the present study is to review studies in problem solving in mathematics to ascertain the studies' general characteristics and determine the research trends on problem solving.

The following research questions were considered:

1. What is the range and frequency of topics being explored in mathematical problem solving?
2. What is the frequency of the sample type with respect to sample size?
3. What are the most frequently research methods?
4. What are the most frequently used data collection tools?
5. What data analysis techniques are employed within the trend?
6. What is the general pattern of the findings of the studies on problem solving in mathematics?

## 3. Methods

In this study, research about mathematical problem solving between the years 2000 and 2020 were analysed in terms of the general characteristics of the studies. The research review conducted in this study is based on a content analysis of research articles that focused on trends on problem solving in mathematics. According to Medelyan (2019), content analysis refers to the categorization, tagging and thematic analysis of qualitative data.

Literature review was conducted through online research articles obtained by using the search terms/ keywords such as "mathematical problem solving, problem solving in mathematics, and research trends in mathematical problem solving". Next, past research articles and topics specifically related to the topic under study were systematically examined.

The articles were selected based on two inclusion criteria: first, articles available online with topics related to the problem solving in mathematics were included in the study. Secondly, this research was limited to the periods 2000 to 2020.

### 3.1 Categories for Trends in Topics Explored

After careful search for the studies and selecting the ones that met the inclusion criteria, a coding protocol was designed and to determine the topical trends, an open-coding technique was used to label and organize the qualitative data in order to identify the various themes and their existing relationships (Medelyan, 2019). According to Khandkar (2009), open coding includes labeling concepts, defining and developing meaningful categories based on their properties and dimensions. In open coding process, there is emergence of concepts from the raw data which is later grouped into conceptual categories for analysis and interpretation.

### 3.2 Categories for Methodological Trends (Research Methods, Data Collection Tools, Data Analysis)

A data set was created with the relevant fields on the research methods (quantitative, qualitative, and mixed), sample type and size, data collection tools as well as data analysis techniques.

## 4. Results and Discussion

This critical review investigated the studies on problem solving in mathematics for the topics explored, research methods, sample type and size, data collection tools and data analysis techniques. For this study, hundred (100) articles related to problem solving in mathematics were examined. The 100 articles were subjected to a content analysis through the determined criteria.

### 4.1 Frequency of Topics Explored in Mathematical Problem Solving

Open coding of this research question yielded fourteen (14) primary topics with 14 sub-topics as viewed in Table 1. The purpose of the research topics formed the basis of the variables generated from the articles reviewed. Most of these were very explicit in the abstracts of the articles sampled. The variables are categorized and coded under the following 14 primary topics respectively: problem solving ability (29%), problem solving teaching (11%), metacognition in problem solving and thinking through problem solving (9%), views about problem solving and problem solving process (7%), problem solving strategies, problem solving creativity and problem solving difficulties (6%), problem solving efficacy (4%), problem solving assessment and problem solving representation and problem solving curriculum (1%).

Table 1: Frequency of Topics in Mathematical Problem Solving (2000 – 2020) (n=100).

| Primary Topic | F | % | Sub-Topics |
|---|---|---|---|
| Problem solving ability | 29 | 29 | Skill |
| Views about problem solving | 7 | 7 | Perception, feeling, belief |
| Problem solving teaching | 11 | 11 | Approach, model, technology use |
| Problem solving strategies | 6 | 6 | |
| Problem solving efficacy | 4 | 4 | Effect |
| Problem solving creativity | 6 | 6 | |
| Metacognition in problem solving | 9 | 9 | |
| Problem solving assessment | 2 | 2 | |
| Thinking through problem solving | 9 | 9 | Learning, comprehension, understanding |
| Problem solving difficulties | 6 | 6 | Errors, complexities |
| Problem solving process | 7 | 7 | Procedure |
| Problem solving representation | 1 | 1 | |
| Problem solving performance | 2 | 2 | |
| Problem solving curriculum | 1 | 1 | |
| TOTAL | 100 | 100% | |

### 4.2 The Most Frequently Employed Research Method

Quantitative, qualitative and mixed methods research were employed in the articles surveyed. Table 2 presents the kinds of research methods researchers preferred within the years under review (2000-2020). The analysis revealed that the researchers mostly conducted qualitative (58%) and quantitative (33%) studies, with mixed methods being (9%). A reason for most of the researchers preferring the qualitative research method over the other could be because it is exploratory in nature and the sample size is typically small enabling the understanding of underlying reasons, opinions, motivations, views and perceptions about problem solving in mathematics. Also, it is useful in uncovering trends in thought and opinions, and dive deeper into the problem in mathematical problem-solving teaching and learning (Carol, 2016; DeFranzo, 2020). This is followed by quantitative research method utilized in studying attitudes, opinions, behaviours, and other defined variables in larger sample population to determine the result. It utilizes measurable data in concluding facts and revealing different research patterns (Carol, 2016). Compared to the other research methods, there were studies with a mixed method. Although mixed method (i) integrates both quantitative and qualitative research designs; (ii) gives an enhanced understanding of the problem and produces more complete and comprehensive evidence; and (iii) strengthens findings, a process called triangulation (Emerald Publishing, 2020).

Table 2: Frequencies of Research Method (2000-2020) (n=100).

| Research method | F | % |
|---|---|---|
| Quantitative | 33 | 33 |
| Qualitative | 58 | 58 |
| Mixed | 9 | 9 |
| TOTAL | 100 | 100% |

### 4.3 Frequency of the Sample Type with Respect to Their Sample Size

Table 3 presents the sample types used in the studies which include: students, teachers, documents, and Mathematics Education professionals. Analysis of the research according to their sample type revealed that, the research mainly investigated the views and opinions of students (67 studies) about problem solving, followed by teachers' beliefs and approach to teaching mathematical problem solving (3 studies). 2 studies investigated problem solving representation in documents such as textbooks, while 1 study investigated mathematics Education professionals.

As regards the sample size, the interval of 1-50 tops the list. A reason for this might be because majority of the researchers conducted qualitative research. Samples in qualitative research tend to be small in order to support the depth of case-oriented analysis that is fundamental to this mode of inquiry. Also, qualitative samples are purposively selected by virtue of their capacity to provide richly textured information relevant to the phenomenon under study, in this case, mathematical problem solving (Vasileiou, Barnett, Thorpe & Young, 2018).

Table 3: Frequency of Sample Types with respect to their Sample Size

| Sample Type | 1-50 | 51-100 | 101-150 | 151-200 | 201-250 | 251-300 | 301 & above | TOTAL |
|---|---|---|---|---|---|---|---|---|
| Students | 36 | 11 | 5 | 7 | - | 2 | 6 | 67 |
| Teachers | 3 | | | | | | | 3 |
| Documents (Textbooks) | 1 | | | | | | | 2 |
| Mathematics Education professionals | 1 | | | | | | | 1 |
| TOTAL | 41 | 11 | 5 | 8 | - | 2 | 6 | 73 |

### 4.4 Data Collection Tools Most Frequently Employed

Table 4 summarises the data collection tools utilized in mathematical problem-solving studies. The analysis shows the test (38%) and interview (26%) as the dominant data collection tools used in mathematical problem-solving research. The dominance of test, being a quantitative data collection tool is not surprising as majority of the researchers investigated mostly the problem-solving ability of students, hence the need for the use of test. A further analysis reveals that in quantitative studies, researchers also used questionnaires (13%) whereas in the qualitative studies, the trend was towards interviews (26%) and observation (17%), in a bid to investigate the views, perceptions, and opinions of the respondents about mathematical problem solving, and to observe the teaching and learning of mathematical problem-solving process in the classroom.

Another trend noted was the use of other data collection tools like document and field note, particularly to detect how mathematical problem solving was being represented in textbooks and assessed in classrooms.

Table 4: Frequency of Data Collection Tools

| Data collection tools | F | % |
|---|---|---|
| Test | 46 | 38 |
| Questionnaire | 16 | 13 |
| Interview | 32 | 26 |
| Observation | 20 | 17 |
| Others | 7 | 5 |
| TOTAL | 121 | 100% |

### 4.5 The Frequency of Data Analysis Technique Employed

Table 5 presents the techniques applied in analysing data in the studies. The most frequently used analysis technique in qualitative research was descriptive analysis (32%), followed by content analysis (15%). A reason for this might be because researchers mostly utilized qualitative research method, thus the need to describe events and categories of information about mathematical problem solving (AECT, 2001). Likewise, in quantitative research, the most frequently used analysis type was mean/standard deviation (11%), followed by frequency and percentage (9%). The studies also utilized inferential statistical analysis method such as: t-test (8%), variance analysis (ANOVA, MANOVA, ANCOVA) (5%), factor analysis (2%) as well as correlation (6%) as parametric test to make assumptions or hypothesize about the parameters of the normal population distribution from which the sample is drawn. While Chi-square (2%), Wilcoxon (2%) and Kruskal Wallis (1%) tests were utilized as the nonparametric tests.

Table 5: The Frequency of Data Analysis Techniques employed

| Data analysis technique | Statistical Tests | F | % |
|---|---|---|---|
| Frequency/Percentage | Descriptive statistics | 11 | 9 |
| Mean/Standard deviation | | 13 | 11 |
| Charts | | 5 | 4 |
| T-test | Parametric test | 10 | 8 |
| ANOVA/MANOVA/ANCOVA | | 7 | 5 |
| Factor Analysis | | 2 | 2 |
| Correlation | | 8 | 6 |
| Chi-square | Nonparametric test | 2 | 2 |
| Wilcoxon test | | 2 | 2 |
| Kruskal Wallis test | | 1 | 1 |
| Content analysis | Qualitative | 18 | 15 |
| Descriptive analysis | | 39 | 32 |
| Discourse analysis analysis | | 2 | 2 |
| Grounded theory analysis | | 1 | 1 |
| TOTAL | | 121 | 100% |

### 4.6 General Pattern of the Findings of the Studies on Problem Solving in Mathematics

The studies shared a common pattern in terms of their findings regardless of the publication date and context (i.e., country) they were conducted. Starting with problem solving ability, some studies revealed that, students' mathematical problem-solving skills were poor, that they don't have the ability to absorb information well, and they lacked experience in doing problems (Meutia, Ikhsan & Saminan, 2020; Rohmali & Sutiarso, 2018). Since problem solving has been proven to have positive effect on problem solving skills improvement and mathematical thinking (Ersoy & Guner, 2015), teachers should pay attention to students' skills in solving non-routine problems and teachers should utilize contextual word problems that are directly related to students' daily life so that they become acquainted with the questions (Meutia, Ikhsan & Saminan, 2020).

As regards problem solving strategies, some studies revealed the significant effect of problem-solving strategies on achievement (Awofala, 2002). Such strategies include making drawing/ diagram, making a table, writing mathematical sentences/ equations, looking for patterns, making a list, using logical reasoning and guess-check strategies (Magno, 2011; Sulak, 2010). But students sometimes still use strategies inappropriate to the problem types (Peterson, McAuliffe & Vermeulen, 2017). Ortiz (2016) stated that a true problem-solving process will allow students to be flexible, intuitive, and creative.

The analysis also revealed some common findings on metacognition in problem solving. Metacognition was defined as the knowledge and awareness about the process of cognition or learning about the mind and how it works. It was stated further that solving mathematical problems require the involvement of metacognition (Arum, Widjajanti & Retnawati, 2019; Trismani & Winarso, 2019). Three metacognitive activity components identified were: awareness, evaluation, and regulation (Hastuti, Nusantara & Susanto, 2016). Consequently, it was found that metacognition has positive influence on mathematical problem solving. The higher metacognition that students have, the better mathematical problem-solving students can do (Izzati & Mahmudi, 2018).

Finally, it was revealed that most learners had difficulty solving mathematical word problems and communicating their thinking through writing (Peterson, McAuliffe & Vermeulen, 2017). For example, Kusumadewi and Retnawati (2020) outlined areas of difficulty found from their study that: students' problem solving ability was low, thus they encounter difficulty in locating keywords from problems because they lack the ability and hence result into guessing the solution without going through the completion process; they have difficulty in modelling the problem into mathematical form; they have difficulty in making decision on how to resolve problems; and have difficulty in counting operations, so that they interpret conclusions that are not appropriate to the problem.

## 5. Conclusion and Recommendation

This paper constitutes a study of trend analysis on mathematical problem solving in journal articles. It provides information on topical issues, as well as the methodological trends and the common findings from the surveyed articles. The study is based on findings on a trend analysis within the period 2000 to 2020, and a content analysis of journal articles available online relevant to problem solving in mathematics. The presented conclusions are based on a thorough search process of the online journal articles using keywords relevant to the topic. The results of the study revealed that problem solving ability (skill) is the major topic in mathematical problem-solving research within the trend period. This is followed by problem solving teaching, thinking through problem solving, metacognition in problem-solving, problem-solving process, and views about problem solving amongst others. Regarding the research methods, it was found that qualitative studies dominated the trend period. The reason might be that it is exploratory in nature, uncovering trends in thoughts and opinions and also enabling the understanding of mathematical problem-solving teaching and learning. The trend regarding the sample type and size revealed that researchers mainly investigated the views and opinions of students about mathematical problem solving and the sample size was typically small with interval 1-50 topping the list.

Furthermore, regarding the data collection tools used, test and interview dominated, while questionnaire was minimal. The dominance of test being a quantitative data collection tool is not surprising as majority of the researchers investigated mostly the problem-solving ability of students requiring the use of test to detect. The trend regarding data analysis technique used showed that researchers in mathematical problem solving rely mainly on qualitative interpretative analysis with a preference for descriptive analysis and content analysis respectively, as the researchers mostly utilized qualitative research method. Likewise, in the quantitative research, the researchers utilized mean/standard deviation and frequency/percentage as data analysis techniques.

In the light of the above findings, the study can provide a base for further studies and a better investigation of research on problem solving in mathematics as it shows the general characteristics of the studies and reveals what has been done so far and the aspects to be researched further in order to improve the teaching and learning of problem solving in mathematics.

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